Math 30-1 Exemplars - SOLUTIONS

Question 1

$g(x)$ shifts $f(x)$ **4 right** and **3 up**

All pts $(x,y) \rightarrow (x+4, y+3)$

So...$(2, 1) \rightarrow (6, 4)$

Vertex of $f(x)$ — $2+4$ — $1+4$

Answer **C**

Question 2

A, B, and C are all on the x-axis

So, transformation must be VERTICAL (stretch or reflection)

A is a horiz stretch – pts move $1/b$ times as far from y-axis. *Not invariant.*

B is a vertical stretch – all pts move a times as far from x-axis. (And A, B, C are all 0 units from it, so they don't move!) ***invariant.***

C is a vertical shift – all pts move k units up, including A, B, and C. *Not invariant.*

D is a horizontal shift – all pts move h units right (or left if h is negative), including A, B, and C. *Not invariant.*

Answer **B**

Question 3

$y = f(x)$ is transformed to

$y = \frac{1}{8}f(-x)$

Isolate y to see vert. stretch

$(-)$ is inside, so horiz. reflection

Answer **B**

Question 4

$f(x)$ has a vertex at $(2, 3)$

$g(x)$ has a vertex at $(-2, 1)$

$g(x)$ is 4 units left of $f(x)$, and 2 units down

$x+2$ inside gives horiz. shift

$+1$ outside gives vert. shift

From x-coord 2 to -2

From y-coord 3 to 1

Answer **4 5 2 8** *Or...* **2 8 4 5**

Question 5

The domain of $y = f(x)$ is $x \geq -2$

After a horiz. str. of 2, the domain of $y = p(x)$ is $\boldsymbol{x \geq -4}$ ← All points move twice as far from the y-axis.

The range of $y = f(x)$ is $y \geq 0$

A horiz.str. does not affect the range *(only x-coords get multiplied by 2)* So the range of $y = p(x)$ stays $\boldsymbol{y \geq 0}$.

The domain of $y = g(x)$ is $x \in R$

A horiz. str. will not change this, so, the domain of $y = q(x)$ is $\boldsymbol{x \in R}$.

The range of $y = g(x)$ is $y \geq -4$

A horiz.str. does not affect the range *(only x-coords get multiplied by 2)* So the range of $y = q(x)$ stays $\boldsymbol{y \geq -4}$.

Answer **3 5 4 7**

Question 6

Reflection about $y = x$ gives the INVERSE function, where all pts $(x,y) \rightarrow (y,x)$

So, the point $A(3, -5)$ becomes $\boldsymbol{(-5, 3)}$.

Reflection about $x = 0$ (the y-axis) gives a horiz. refl., where $(x,y) \rightarrow (-x,y)$

So, the point $A(3, -5)$ becomes $\boldsymbol{(-3, -5)}$.

Reflection about $y = 0$ (the x-axis) gives a vert. refl., where $(x,y) \rightarrow (x, -y)$

So, the point $A(3, -5)$ becomes $\boldsymbol{(3,5)}$.

Answer **5 3 1**

Question 7

If a graph is of a FUNCTION there will be 1 (and only 1) y value assigned for each x in the domain. (So graph 3 is NOT a function since when $x = 0$ it can be seen that there are three different y values. Look up "*Vertical Line Test*" for more info.)

When graphing an inverse function, all pts $(x,y) \rightarrow (y,x)$. So for its inverse to be a function we are looking for graphs where each y in the range maps from 1 (and only 1) x. (Look up "*Horizontal Line Test for inverses*" for more info)

For graph 2 we can see that when $y = 0$ there are 3 different x values, so the inverse of graph 2 will NOT be a function.

However for graphs 1 and 3 there are no scenarios where a particular y has more than one associated x. So the inverses of these graphs ARE FUNCTIONS! (even though graph 3 is not a function itself!)

Answer **B**

Question 8

For the range of a graph we need to consider the lowest and highest points.

For example $f(x)$ has a lowest point where $y = -4$, and a highest point where $y = 8$, so its range is $[-4, 8]$.

The lowest point on $(f-g)(x)$ occurs when the lowest value of f *($y = -4$)* is subtracted by the highest value on g, ($y = 2$) ***That is, the lowest pt is $y = -6$.***

The highest point occurs when the highest value of f *($y = 8$)* is subtracted by the lowest value on g, *($y = -4$)* ***That is, the highest pt is $y = 12$.***

Answer **C**

Question 9

For the described transformations, all pts $(x,y) \rightarrow (4x, -\frac{1}{3}y)$ Note, we do NOT use the reciprocal of the horiz. str. factor unless we are writing an equation, in this case it would be $g(x) = \frac{1}{3}f(\frac{1}{4}x)$.

So, the point $(-3, 6)$ becomes $\boldsymbol{(-12, 2)}$

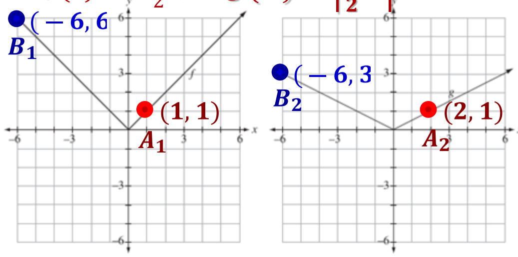
$4*(-3)$ — $-\frac{1}{3}*6$

Answer **D**

Question 10

Option 1 – Horizontal Stretch

Consider two points that line up horizontally – such as A_1 and A_2 as shown below. Point A_2 is two times as far from the y-axis as A_1.
For a horiz. str. by factor of 2, the equation is $g(x) = f(\frac{1}{2}x)$ or $\boldsymbol{g(x) = \left|\frac{1}{2}x\right|}$ ⬅



Option 2 – Vertical Stretch

Consider two points that line up vertically – such as B_1 and B_2 as shown above. Point B_2 is half as far from the x-axis as B_1. For a vert. str. by factor of 1/2, the equation is $g(x) = \frac{1}{2}f(x)$ or $\boldsymbol{g(x) = \frac{1}{2}|x|}$ ⬅

Question 11

Re-arrange: $g(x) = f(x+5) + 2$, which shows a horiz. translation 5 units left and a vertical translation 2 units up.

➔ All pts $(x, y) \to (x-5, y+2)$ So, $(3, 1) \to (\mathbf{-2, 3})$

Answer **B**

Question 12

You must factor the inside to get the hor. shift right!

Re-arrange: $y = \sqrt{-\frac{1}{2}(x+8)}$, which shows a horiz. reflection, a horiz. str. factor of 2, and a horiz. translation 8 units left.

Answer | 2 | 4 | 7 | |

**numbers can be in any order*

Question 13

➔ All pts $(x, y) \to (-\frac{1}{2}x, 2y+3)$

So, $(-10, 0) \to (\mathbf{5, 3})$

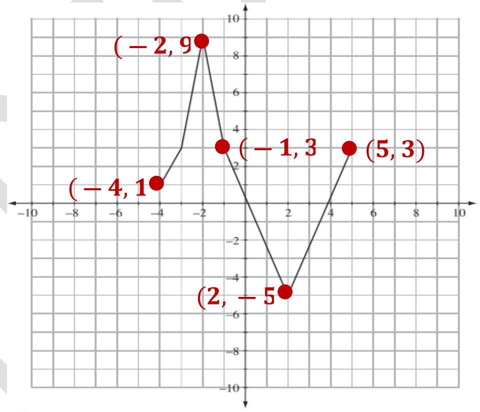
$-\frac{1}{2} * (-10)$ $2 * (0) + 3$

$(-4, -4) \to$

$(2, 0) \to (\mathbf{-1, 3})$

$(4, 3) \to (\mathbf{-2, 9})$

$(8, -1) \to (\mathbf{-4, 1})$



Question 14

To determine an inverse function, we:

Re-write using y instead of $f(x)$

$y = 2x - 3$

Interchange x and y

$x = 2y - 3$

Isolate y

$x + 3 = 2y$ ➡ $\frac{2y}{2} = \frac{x}{2} + \frac{3}{2}$

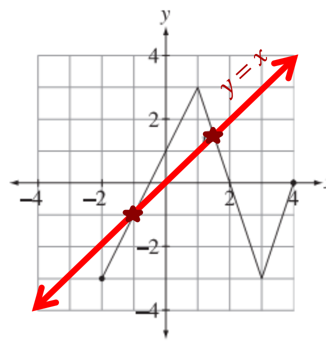
➡ $\boldsymbol{g(x) = \frac{1}{2}x + \frac{3}{2}}$ *Answer* | 1 | 3 | | |

Question 15

When graphing an inverse function, all pts $(x,y) \to (y,x)$.

So, any invariant point would occur where the x and y coordinates are the same. *That is, on the line $y = x$*

To identify invariant pts, draw the line $y = x$ on top of the graph of f. *Answer* **B**



Question 16

At $x = 3$, the value of f is 6, and the value of g is 1.

So, the value of $f(3)/g(3)$ is $\frac{6}{1} = \mathbf{6}$

Answer | 6 | | | |

Question 17

At $x = 5$, the value of f is 55, and the value of g is $1/3$. ← $(5)^2 + 6(5)$

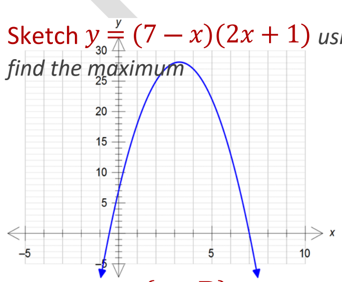
So, the value of $h(5) = g(5) + f(g(5))$

$= \frac{1}{3} + (\frac{1}{3})^2 + 6(\frac{1}{3})$

Answer | 2 | . | 4 | 4 |

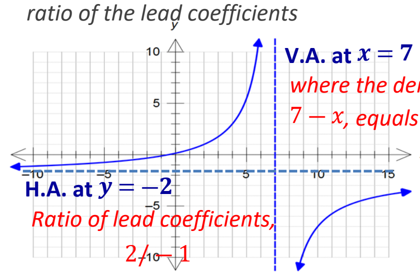
Question 18

(a) Sketch $y = (7-x)(2x+1)$ *using calc, find the maximum*



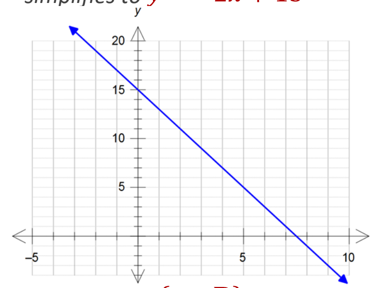
Domain: $\{x \in \mathbb{R}\}$
Range $\{y \leq 28.125\}$

(b) Sketch $y = (2x+1)/(7-x)$ *using calc, recall that horiz. asymp. occurs at the ratio of the lead coefficients*



Domain: $\{x \neq 7\}$
Range $\{y \neq -2\}$

(c) Sketch $y = 2(7-x) + 1$ *which simplifies to* $y = -2x + 15$



Domain: $\{x \in \mathbb{R}\}$
Range $\{y \in \mathbb{R}\}$

Question 19

$k(x) = 2[(\sqrt{x-1})^2 + 3] - 5$ *simplifies to…*

$= 2(x - 1 + 3) - 5$

$= 2(x + 2) - 5$

$= 2x + 4 - 5$

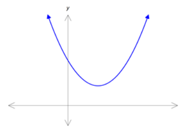
$= \mathbf{2x - 1}$

Answer 2 1

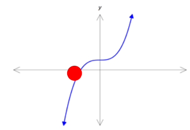
Question 20

The domain of a rational function will be $x \in \mathbb{R}$ if the "bottom" (denominator) can never be 0. Given that $b > 1$…

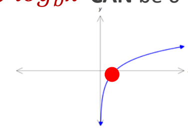
❷ $x^2 + b$ can never be 0



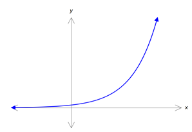
❸ $x^3 + b$ **CAN** be 0



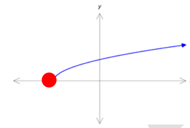
❹ $log_b x$ **CAN** be 0



❺ b^x can never be 0



❸ $\sqrt{x+b}$ **CAN** be 0



Answer 2 5

Question 21

First note that $f(3) = 5$, then note that $g(5) = 8$

From the pt$(3, 5)$

Answer **B**

Question 22

Simplify: $h(x) = \dfrac{x^2 - 7x}{x - 2} + \dfrac{2x^2 + x}{x - 2}$

$= \dfrac{x^2 - 7x + 2x^2 + x}{x - 2}$

$= \dfrac{3x^2 - 6x}{x - 2}$ ➡ $= \dfrac{3x(x-2)}{x - 2}$ ➡ $= \mathbf{3x}$

Question 23

Given: $g(3) = 4$, and $f(g(3)) = 8$ ➡ So, $f(4) = 8$

But hey, this is, 4

So, the corresponding point is $(4, 8)$

Question 24

Simplify: $h(x) = \dfrac{(\sqrt{x-3})^2}{(\sqrt{x-3})^2 - 25}$

$= \dfrac{x - 3}{x - 3 - 25}$

$= \dfrac{x - 3}{x - 28}$

Can't $\sqrt{\ }$ negatives!

For the domain of $h(x)$, we must consider:

- Before simplifying, $h(x)$ had a radical component, so $\mathbf{x \geq 3}$
- The domain of the simplified func. / can't divide by 0 So, $\mathbf{x \neq 28}$

Combined, we get $x \geq 3, x \neq 28$

Note, we do not need to consider the domain of $g(x)$ on its own, that is $x \neq \pm 5$, because the inputs for $g(x)$ are values of f, not "x".

Answer 4

Question 25

Reflection in the line $y = x$ means find the **inverse**.

$x = 3^{y+2}$ Switch x and y

$y + 2 = log_3(x)$ Convert to log form, and solve for y

$y = log_3(x) - 2$

Answer **A**

Question 26

$\dfrac{(5^3)^{x(x+1)}}{5^{3x-4}} = (5^2)^{x-5}$ Re-write all terms using *a common base*

$\dfrac{5^{3x^2+3x}}{5^{3x-4}} = 5^{2x-10}$ Apply exponent rules to simplify

$5^{(3x^2+3x)-(3x-4)} = 5^{2x-10}$

Set exponents equal

$3x^2 + 3x - 3x + 4 = 2x - 10$

$3x^2 - 2x + \mathbf{14} = 0$

Answer **C**

Question 27

$(2^3)^{3x+4)} = (2^2)^{x-9}$ Re-write all terms using a *common base*

$2^{9x+12} = 2^{2x-18}$ Apply exponent rules to simplify

$9x + 12 = 2x - 18$ Set exponents equal

$7x = -30$ ➡ $\mathbf{x = -\dfrac{30}{7}}$

Answer **B**

Question 28

$m log_p n = q - 5$ ➡ $log_p n^m = q - 5$ ➡ $p^{q-5} = n^m$

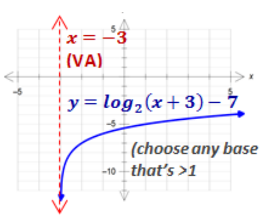
Isolate log term   Apply log law

Answer **B**

Question 29

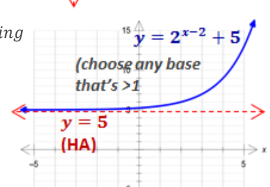
$f(x)$ has a V.A. at $x = -3$

Think: You can't log 0 or negatives, so the domain of f is $x + 3 > 0$ ➔ $x > -3$



$g(x)$ has a H.A. at $y = 5$

Think: The power term $2^{anything}$ can never be 0 or a negative (must be +), so the +5 after means the range of g is $y > 5$

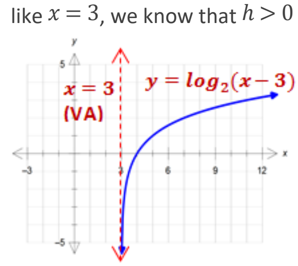


Answer **B**

Question 30

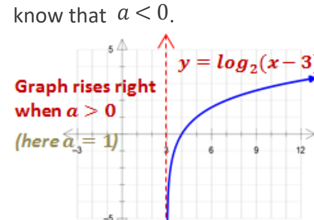
$\mathbf{h > 0}$

Since the V.A. is positive (for example, it could be something like $x = 3$, we know that $h > 0$



$\mathbf{a < 0}$

Since the graph "falls right", we know that $a < 0$.



Answer **A**

Question 31

$L_{noise\ maker} = 127db$
(Given)

$L_{difference} = 10log5000$
(Difference between loudness of noise maker and lawn mower)
$\approx 164\ db$

So, loudness of the lawn mower is:

Answer $\approx 164\ db - 127\ db$

C $\approx \mathbf{90\ db}$

Question 32

$log\left(\frac{2sinx}{sin2x}\right)$

$= log\left(\frac{2sinx}{2sinxcosx}\right)$ Apply double angle identity for $\boldsymbol{sin}$

$= log\left(\frac{1}{cosx}\right)$ Simplify / cancel

$= log1 - log\ (cosx)$ Apply log law

$= 0 - log\ (cosx)$ *Answer* **D**

Question 33

$log(3^{2x+1}) = log\left(\frac{1}{5}\right)^{x-3}$ "log both sides" to solve exp. eqn. where no common base is possible

$(2x+1)log(3) = (x-3)log\left(\frac{1}{5}\right)$

$2x(log3) + log3 = x(log0.2) - 3log\ (0.2)$

$log3 + 3log0.2 = x(log0.2) - 2x(log3)$

$x(log0.2 - 2log3) = log3 + 3log0.2$

$x = \frac{(log3 + 3log0.2)}{(log0.2 - 2log3)}$ $\boldsymbol{x \approx 0.98}$

Question 34

$a^3 = 8$ $4^{3/2} = b$

Convert each to exp. form

$a = 2,\ b = 8$ Evaluate

$a = \sqrt[3]{8}$ $b = \sqrt[2]{4^3}$

Now find:

$log_2 8 + log_8 2$

$= 3 + 1/3$

Since $2^3 = 8$ Since $8^{1/3} = 2$

Answer **3 . 3 3**

Question 35

$log_7[(x+1)(x-5)] = 1$ Combine to a single log

$7^1 = x^2 - 5x + 1x - 5$ Convert to exp. Form (and expand)

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$ Solve resulting quadratic equation

$x = 6$ or $x = -2$

Check each equation by subst. into the original equation

$log_7(6+1) + log_7(6-5) = 1$

$log_7(7) + log_7(1) = 1$

$1 + 0 = 1$ ✓

$log_7(-2+1) + log_7(-2-5) = 1$

$log_7(-1) + log_7(-7) = 1$ ✗ can't log negatives – sol. is **EXTRANEOUS**

➡ $\boldsymbol{x = 6}$

Question 36

Use $y = ab^{\frac{t}{b}}$

end amount; initial amount; mult. growth factor; period of time for growth factor (here "1"); ...in how long

Solve for "b":

$32\ 450 = 15000b^8$

$2.16333 = b^8$ *Isolate the power term*

$b = (2.16333)^{\frac{1}{8}}$ *take the eighth root of both sides*

$b = 1.1012$ *THINK:* $b = 1 + growth\ rate$

growth rate is right here!

ANSWER: **10.1%** *Answer* **C**

Question 37

$log_a b^{\frac{1}{3}}$ *Simplify to isolate* $log_a b$

$= \frac{1}{3} log_a b$

Given: This is 1.26

$= \frac{1}{3}(1.26)$ *Answer* **0 . 4 2**

≈ 0.42

Question 38

$y = ab^{\frac{t}{p}}$

$6000 = a(1.04)^{\frac{3}{0.5}}$

$a = \frac{6000}{(1.04)^6}$ $\boldsymbol{a = \$4742}$ *answer*

$y = 6000$ *(end amount)*

$a =$ ***wanted*** *(start amount)*

$b = 1 + \frac{0.08}{2}$ *(mult. growth factor)*

$t = 3$ *(how long)*

$p = 0.5$ *(since compounded semi-annually)*

Question 39

Since -1 and -2 are zeros, $P(-1) = 0$ and $P(-2) =$ (

$2(-1)^5 + 3(-1)^4 - 10(-1)^3 - 21(-1)^2 + k(-1) = 0$

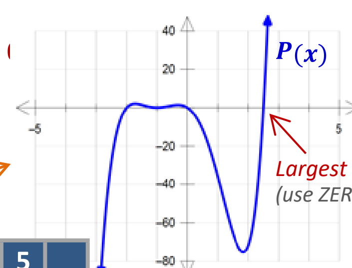
Use either to solve for k

$-2 + 3 + 10 - 21 - k = 0$

$-10 = k$

So, $P(x) = 2x^5 + 3x^4 - 10x^3 - 21x^2 - 10x$

GRAPH to see largest zero



Largest zero is 2.5 (use ZERO on calc!)

Answer **2 . 5**

Question 40

Since $P\left(-\frac{2}{3}\right) = 0$, one of the factors would be $\left(x + \frac{2}{3}\right)$ OR $(3x+2)$

Mult both terms by 3

Since $P(0) = 12$ the constant term is 12. (For example, think of the function $y = x^3 - 3x + 12$)

Answer **B**

Question 41

Since $P\left(-\frac{2}{3}\right) = 0$, one of the factors would be $\left(x + \frac{2}{3}\right)$ OR $(3x+2)$

Mult both terms by 3

Since $P(0) = 12$ the constant term is 12. (For example, think of the function $y = x^3 - 3x + 12$)

Answer **B**

Question 42

Fully factor: **Step ❶** Potential zeros: $\pm 1, \pm 3$ **Step ❷** Test: $P(1) = 2(1)^3 - 3(1)^2 - 10(1) + 3$, which $\neq 0$ Similarly $P(-1) \neq 0$, but $P(3) = 2(1)^3 - 3(1)^2 - 10(1) + 3$, which $= \mathbf{0}$ So, $(x-3)$ is a factor. **Step ❸** Synthetic division to find $(2x^3 - 3x^2 - 10x + 3) \div (x-3)$

-3	2	-3	-10	3
	↓	-6	-9	3
	2	3	-1	0

Step ❹ Synth division gives remaining factor of : $3x^2 - 10x + 3$, which doesn't factor. So use quad formula to find remaining roots.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ➡ $x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$ ➡ $x = \frac{-3 \pm \sqrt{17}}{4}$

Answer **1 7**

Question 43

Since the zeros of the function are –3, 1, and 5 (multiplicity 2), the equation that represents the function is $f(x)=\frac{-1}{a}(x+3)(x-1)(x-5)^2$. Use the y-intercept (0, 5) to find the leading coefficient.

$$5=\frac{-1}{a}(0+3)(0-1)(0-5)^2$$

$$5=\frac{-1}{a}(-75)$$

$$-\frac{1}{15}=\frac{-1}{a}$$

$$\therefore f(x)=-\frac{1}{15}(x+3)(x-1)(x-5)^2$$

Question 44

Since $x+2$ is a factor, the remaining factor can be found by dividing $P(x)\div(x+2)$

-2	4	-1	-8	0	-40
		-8	18	-20	40
	4	-9	10	-20	0 ← Remainder

$4x^3-9x^2+10x-20$

Answer **C**

Question 45

Since $x+2$ is a factor, $P(-2)=0$

$(-2)^3+3(-2)^2+k(-2)+4=0$

$-8+12-2k+4=0$

$8=2k$

$k=4$

Answer: 4

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Question 46

$P(-1)=2(-1)^4+3(-1)^3-17(-1)^2-27(-1)-9=0\rightarrow(x+1)$ is a factor

1	2	3	-17	-27	-9
	↓	2	1	-18	-9
	2	1	-18	-9	0

$Q(x)=2x^3+x^2-18x-9$

$Q(3)=2(3)^3+3^2-18(3)-9=0\rightarrow(x-3)$ is a factor

-3	2	1	-18	-9
	↓	-6	-21	-9
	2	7	3	0

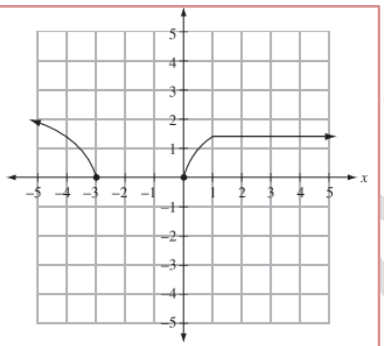
$2x^2+7x+3=(2x+1)(x+3)$

$\therefore P(x)=(x+1)(x-3)(2x+1)(x+3)$

Question 47

D: $(-\infty,-3]$ or $[0,\infty)$

R: $[0,\infty)$



Question 48

Point **C**$(4,\ 90)$ transforms to $(4,\sqrt{90})$

But $\sqrt{90}>9$ (Max value of range)

So point C is not possible on $f(x)$

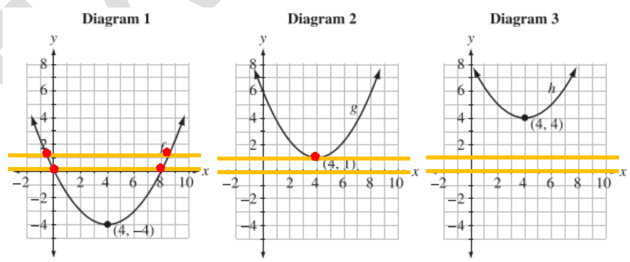
Answer **C**

Question 49

Draw horizontal lines at $y=0$ and $y=1$

(Invariant points where $f(x)=0$ or 1)

Answer: 4 1 0



4 invariant points **1 invariant point** **0 invariant points**

Question 50

Invariant points where $f(x)=\mathbf{0}$ or $\mathbf{1}$

$\frac{1}{2}x-3=\mathbf{0}$ $\quad$ $\frac{1}{2}x-3=\mathbf{1}$

$\frac{1}{2}x=3$ $\quad$ $\frac{1}{2}x=4$

$x=6$ $\quad$ $x=8$

Pt $(\mathbf{6},\mathbf{0})$ $\quad$ Pt $(\mathbf{8},\mathbf{1})$

Answer: 6 0 8 1

OR

8 1 6 0

Question 51

For x-intercept set $y=\mathbf{0}$

$\mathbf{0}=-2\sqrt{x+4}+3$

$2\sqrt{x+4}=3$

$\sqrt{x+4}=3/2$ *Sq root both sides*

$x+4=9/4$

$x=-7/4$

Answer: 1 . 7 5

Question 52

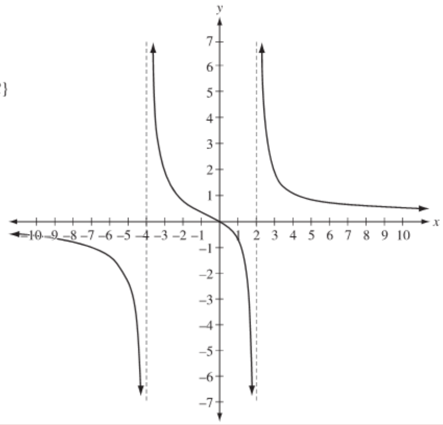
$y = \frac{3x}{(x+4)(x-2)}$

D: $\{x \mid x \neq -4, 2, x \in R\}$

x-intercept: 0

y-intercept: 0

Equation of vertical asymptotes: $x = -4$ and $x = 2$



$y = \frac{x+3}{x^2-9}$

Possible solution:

$y = \frac{x+3}{(x+3)(x-3)}$

$y = \frac{1}{x-3}$

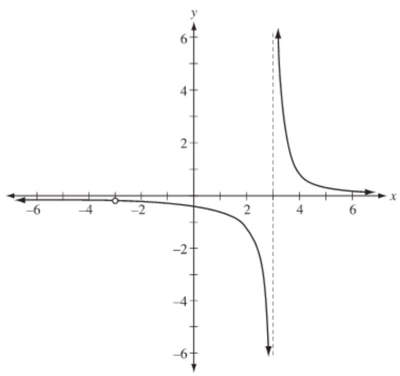
D: $\{x \mid x \neq \pm 3, x \in R\}$

No x-intercept

y-intercept: $-\frac{1}{3}$

Equation of vertical asymptote: $x = 3$

There is a point of discontinuity at $x = -3$.



Question 53

Consider graph of $y = \frac{1}{x}$ *V.A. at $x = 0$*   *H.A. at $y = 0$*

Shifted 1 unit left & 3 units up

V.A. at $x = -1$ H.A. at $y = 3$

Answer: 2 8

Question 54

$f(x) = \frac{(3x+1)(x-4)}{(x-4)(x+4)}$ *Factor*

$f(x) = \frac{(3x+1)}{(x+4)}$ P.D. at $x = 4$ (from factor that cancels)

$f(4) = \frac{3(4)+1}{(4)+4}$ subst. x-coord of PD into simplified form of $f(x)$

$= \frac{13}{8}$

Answer D

Question 55

There is a point of discontinuity when $x = 7$.

$\therefore f(x) = 2x - 1$, for $x \neq 7$

$y = 2(7) - 1$

$y = 13$

So the point of discontinuity is (7, 13).

Question 56

$f(x) = \frac{(3x-2)(2x+3)}{(1-x)(3x-2)}$ *Simplify*

$f(x) = \frac{2x+3}{1-x}$

V.A. at $x = 1$ For H.A., refer back to original function.

$f(x) = \frac{6x^2 \ldots}{-3x^2 \ldots}$ H.A. at ratio of lead coefficients. (Same degree top / bottom)

H.A. at $y = -2$ *Answer* D

Question 57

$\frac{3\pi}{2} = \frac{20.0m}{r}$ $\theta = \frac{3}{4} * 2\pi$

$\frac{3\pi}{2} r = 20.0m$

$r = \frac{20}{\frac{3\pi}{2}}$

$r \approx 4.2$

Answer: 4 . 2

Question 58

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$\frac{\pi}{6} + 2\pi$ Add or subtract 1 rotation (2π) from 30°, that is $\frac{\pi}{6}$.

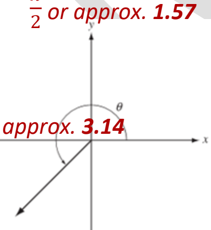
$= \frac{\pi}{6} + \frac{12\pi}{6}$

$= \frac{13\pi}{6}$ Add another rotation…

$\frac{13\pi}{6} + \frac{12\pi}{6} = \frac{25\pi}{6}$

Answer D

Question 59



$\frac{\pi}{2}$ *or approx.* ***1.57***

π *or approx.* ***3.14***

$\frac{3\pi}{2}$ *or approx.* ***4.71***

So, θ must be between 3.14 and 4.71

Answer C

Question 60

$x^2 + y^2 = 1$ Equation of Unit Circle

$(k)^2 + (0.6)^2 = 1$

$k^2 = 1 - 0.36$

$k^2 = 0.64$ Sq root both sides

$k = 0 \pm 0.8$ k is (-) since P is in quad II

$\boldsymbol{k = -0.8}$

$sec\theta = \frac{hyp}{adj}$

(since unit circle)

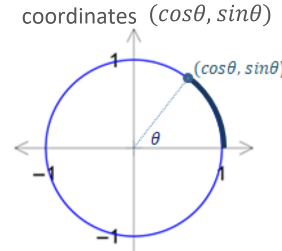
$sec\theta = \frac{1}{-0.8}$

$\boldsymbol{sec\theta = -\frac{5}{4}}$

Answer A

Question 61

Any pt on unit circle has coordinates $(cos\theta, sin\theta)$



$P(cos70°, sin70°)$

$P(0.34, 0.94)$

Answer: 3 4 9 4

Question 62

Any pt on unit circle has coordinates $(cos\theta, sin\theta)$

$\frac{12}{13} \div \frac{-5}{13}$ $\quad 1 \div \frac{12}{13}$ $\quad 1 \div \frac{-5}{13}$ $\quad 1 \div \frac{-12}{5}$

$\sin\theta = \frac{12}{13};\ \cos\theta = -\frac{5}{13};\ \tan\theta = -\frac{12}{5};\ \csc\theta = \frac{13}{12};\ \sec\theta = -\frac{13}{5};\ \cot\theta = -\frac{5}{12}$

Question 63

In Quad II, so tan is (-)

$tan\theta = \frac{sin\theta}{cos\theta}$

$= \frac{-2cos\theta}{cos\theta}$

$= -2$

Now,

$tan2\theta = \frac{2tan\theta}{1 - tan^2\theta}$

$= \frac{2(-2)}{1-(-2)^2}$

$= \frac{-4}{-3}$

Answer **A**

Question 64

sin is (+) and **tan** is (-), so we're in Quad II. *And x is (-)*

$x^2 + (7/10)^2 = 1$ Equation of unit circle

$x^2 = 1 - \frac{49}{100}$ $\quad x^2 = \sqrt{51/100}$

$x^2 = 51/100$ *Answer* **A**

Question 65

A *is at* $\frac{\pi}{6}$

B *is at* $\frac{3\pi}{4}$

$\theta = \frac{3\pi}{4} - \frac{\pi}{6}$

$= \frac{9\pi}{12} - \frac{2\pi}{12}$

$= \frac{7\pi}{12}$

Answer **7 1 2**

Question 66

$= sin\left(\frac{11\pi}{6}\right) + cos\left(\frac{7\pi}{4}\right)$

coterminal

$= -\frac{1}{2} + \frac{\sqrt{2}}{2}$ $\quad = \frac{-1+\sqrt{2}}{2}$

Answer **A**

Question 67

Since θ is a second quadrant angle, $x = -\sqrt{39}$.

Therefore, $\tan\theta = -\frac{5}{\sqrt{39}}$ or $\tan\theta = -\frac{5\sqrt{39}}{39}$.

Question 68

$x - 20° = cos^{-1}(1/2)$

$x - 20° = 60°$

$x° = 80°$

Answer **8 0**

Question 69

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$

$30°$ $\quad 150°$ $\quad 210°$ $\quad 330°$

*Statement **1** is false*

*Statement **2** is false*

(adding 2π to $\frac{\pi}{6}$ does not give the next angle, $\frac{5\pi}{6}$)

*Statement **3** is **true***

$sin\frac{7\pi}{6} = \frac{-1}{2}$ $\quad cos\frac{5\pi}{6} = \frac{-\sqrt{3}}{2}$

Answer **3**

Question 70

If $csc\theta = 2/\sqrt{3}$, then $sin\theta = \sqrt{3}/2$

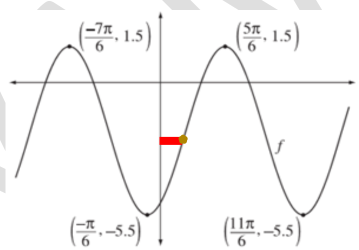
sin is (+) in I or II

$cot\theta = \frac{cos\theta}{sin\theta}$ $\quad$ From unit circle, when y-coord is $\sqrt{3}/2$

$= \frac{\pm 1/2}{\sqrt{3}/2}$ $\quad = \pm\frac{1}{2} \div \frac{2}{\sqrt{3}}$ $\quad = \pm\frac{1}{\sqrt{3}}$

Answer **C**

Question 71



c is midway between the min $(\frac{-\pi}{6})$ and the max $(\frac{5\pi}{6})$

$c = \frac{-\frac{\pi}{6} + \frac{5\pi}{6}}{2}$ $\quad c = \approx 1.05$

Answer **A**

Question 72

$b = \frac{2\pi}{period}$

$b = \frac{2\pi}{8}$ *4 laps in 32 mins, so 1 lap in 8mins*

$b = \frac{\pi}{4}$

$d =$ *median dist from P lot*

$d = 5m + 195m$

$d = 200m$

Answer **D**

Question 73

$y = sin\left[3\left(x + \frac{\pi}{3}\right)\right] + 7$

Factor out the b value to see the phase shift

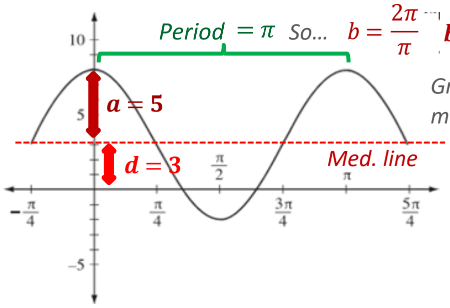
phase shift $= \frac{\pi}{3}$

$period = \frac{2\pi}{b}$

$\boldsymbol{period = \frac{2\pi}{3}}$

Answer **C**

Question 74



So... $b = \frac{2\pi}{\pi}$ $\quad \boldsymbol{b = 2}$

Graph "starts" at its max, so $\boldsymbol{c = 0}$

Answer **5 2 0 3**

Question 75

*In **step 2** both sides are divided by "cosθ"*

*This is **not allowed** as it's deletes solutions*

Answer **A**

Question 76

NPVs where we'd divide by 0

$1 - cos^2\theta \neq 0$ $\quad sin^2\theta \neq 0$

Identity: This is also $sin^2\theta$

$sin^2\theta \neq 0$ *Sq root both sides*

$sin\theta \neq 0$

$\theta \neq 0, \pi, 2\pi, etc$

Answer **A**

Question 77

Left Side:

$$= \frac{\sin\left(\frac{2\pi}{3}\right)}{1 - \cos\left(\frac{2\pi}{3}\right)}$$

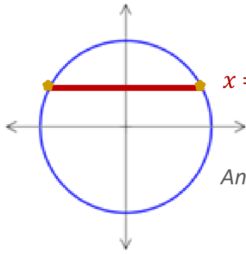
$$= \frac{\frac{\sqrt{3}}{2}}{1 - \left(-\frac{1}{2}\right)}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} \Rightarrow = \frac{\sqrt{3}}{2} * \frac{2}{3} \Rightarrow = \frac{\sqrt{3}}{3}$$

Answer **D**

Question 78

All is good to step 6. There are no solutions for $\sin x = -3$, *but* $\sin = \frac{1}{2}$ *has* ***two*** *solutions*



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Answer **D**

Question 79

$2\cos\theta(\cos\theta + 1) = 0$

$\cos\theta = 0$ $\quad$ $\cos\theta = -1$

$\theta = \cancel{90°}, 270°$ $\quad$ $\theta = 180°$

Note: $90°$ *is not within given solution domain*

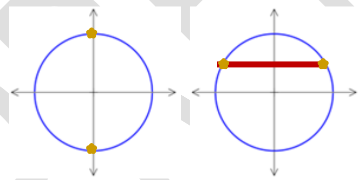
$\theta = 180°$ *or* $270°$

Question 82

Factor to solve:

$\cos\theta(2\sin\theta - 1) = 0$

$\cos\theta = 0$ $\quad$ $\sin\theta = 1/2$



$\theta = 90°, 270°$ $\quad$ $\theta = 30°, 150°$

General Solution options:

- $90°$ *then every* $180°$
- $30°$ *then every* $360°$
- $150°$ *then every* $360°$

- $90°$ *then every* $360°$
- $30°$ *then every* $120°$

Answer **D**

Question 81

$2\cos^2 x + \sin x - 1 = 0$

$2(1 - \sin^2 x) + \sin x - 1 = 0$

$2 - 2\sin^2 x + \sin x - 1 = 0$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 1$

$\left\{-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2}\right\}$

Question 80

$\cot^2\theta = 1$

$\cot\theta = \pm 1$

$\tan\theta = \pm 1$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$, etc.

Therefore, the general solution is $\theta = \frac{\pi}{4} + \frac{n\pi}{2}, n \in I$.

Question 83

$\theta = -109°$ and $-30°$

Enter the following function into the calculator.

$y_1 = \left(2 - \frac{\sqrt{3}}{\cos x}\right)\left(\frac{1}{\cos x} + 3\right)$

A window that could be used is x: [–180, 0, 30], y: [–5, 5, 1].

The x-intercepts are the solutions to the original equation.

Question 84

$x^2 + (-2)^2 = 7^2$

$x^2 = \sqrt{45}$

$x = \pm 3\sqrt{5}$

Since θ is in Quadrant IV, $x = 3\sqrt{5}$ and $\cos\theta = \frac{3\sqrt{5}}{7}$.

$$\cos\left(\theta - \frac{2\pi}{3}\right) = \cos\theta\cos\frac{2\pi}{3} + \sin\theta\sin\frac{2\pi}{3}$$

$$= \left(\frac{3\sqrt{5}}{7}\right)\left(-\frac{1}{2}\right) + \left(-\frac{2}{7}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{-3\sqrt{5}}{14} - \frac{2\sqrt{3}}{14}$$

$$= \frac{-3\sqrt{5} - 2\sqrt{3}}{14}$$

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Question 85

$= 3 * 3 * 3 * 3 * 3 * 3$

Six games

$= 3^6$ *Answer* **A**

Question 86

Alberta:

$23 * 23 * 23 * 10 * 10 * 10$

Letters $\quad$ *Digits*

Ontario:

$23 * 23 * 23 * 23 * 10 * 10 * 10$

Letters $\quad$ *Digits*

Answer **2 3**

(Ontario has one extra "23")

Question 87

$5 \quad * 21 \quad * 20$

- 5: *1, 3, 5, 7, or 9*
- 21: ***26*** *letters –* ***5*** *(vowels)*
- 20: ***26*** *letters –* ***5*** *(vowels)* ***–1*** *(no repetition)*

Answer **2 1 0 0**

Question 88

No arranging / designating positions, so it's a ***COMB***

"At most" two doctors means ***0*** *or* ***1*** *or* ***2*** *doctors. (Three cases)*

Answer **B**

Question 89

4 actors or 5 actors or 6 actors… (3 cases)

$= {}_9C_4 * {}_7C_2 + {}_9C_5 * {}_7C_1 + {}_9C_6 \quad = 3612$

Answer **D**

Question 90

# of terms = 1 + degree

$a - 5 = 6$ $\quad$ $\boldsymbol{a = 11}$

$(2x+3)^6$ *First term has coefficient:*

${}_6C_0 * 2^6$

***coeff* = 64**

Answer **A**

Question 91

$t_{5+1} = {}_8C_5 * (x^3)^{8-5}\left(\frac{1}{2x^2}\right)^5$

$t_6 = 56(x^3)^3\left(\frac{1}{2x^2}\right)^5$

$= 56 * x^9 * \frac{1}{32x^{10}}$

$= \frac{56x^9}{32x^{10}}$

$= \frac{7}{4x}$

Answer **B**

Question 92

On formula, coeff. of "x" (first term) is $n - k$. Since we need power of first term to be 4, and $n = 10$, that means $k = 6$.

$t_{6+1} = {}_{10}C_6(3a)^{10-6}(-b^2)^6$

$= 210 * 81a^4 * (b^{12})$

$= 17010a^4b^{12}$

Answer **D**

Question 93

The exponent of the variable for a constant term must be zero; i.e., a^0.

${}_8C_4(2a)^4\left(\frac{1}{a}\right)^4$

$70(16a^4)\left(\frac{1}{a^4}\right)$

1 120

Therefore, the constant term is 1 120.

Question 94

"n" is one less than the # of terms

OR: $\#\ of\ terms = n + 1$

$\boldsymbol{n = 5}$

Constant term is LAST TERM in expansion:

$t_{5+1} = {}_5C_5 * x^0 * 4^5$

$t_6 = 1024$

Answer **A**